1 Finding the Parameters of the Ellipse: Algebra 2

Figure 1: Elliptical shield. Let’s call the coordinates of the left spaceship and the right spaceship \((x_1, y_1), (x_2, y_2)\), respectively.

Here’s the set up:

1. You have the coordinates of the foci (the location of the two spaceships).
2. For now, the spaceships will have the same height.
3. The coordinates are \((x_1, y_1), (x_2, y_2)\).

How do we figure out what the ellipse looks like from just this? It turns out that you cannot. But that doesn’t mean we can’t draw an ellipse. It just means that the just having the foci isn’t enough to describe everything about an ellipse. Let’s see why.

Figure 2: Ellipses: two different ellipses but with the same foci length. The length of the major radius is on the left side of the ellipse while the focus, \(F\), is listed on the right side.

1.1 Definition of an Ellipse

One way to think about an ellipse is to think about it similar to what we thought about a circle.

A circle is the set of points such at every point is equidistant from the center.

\[
(x - x_c)^2 + (y - y_c)^2 = r^2 \tag{1}
\]

Or it could be rewritten as

\[
\frac{(x - x_c)^2}{r^2} + \frac{(y - y_c)^2}{r^2} = 1 \tag{2}
\]

An ellipse is the set of points such that the sum of the distances from each foci to that point is the same for all points. More specifically, the sum of the distances is equal to twice the length of the major radius, or \(2a\).

\[
\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{b^2} = 1 \tag{3}
\]
Great. But what is the length of the major radius? To make an analogy, it’s like asking what the radius of the circle is if you are only given the center. There is no one unique answer, but that’s perfectly fine. You can choose whatever radius you want.

The same can be said about the ellipse with a few constraints. The major radius has to be longer than the length of the focus, but nothing other than that. As seen in Fig. 2 an ellipse can have two really different shapes even though the length of the foci are the same.

1.2 Initial Questions

Answer the following questions. It may be helpful to play with the parameters of an ellipse (seen here: [http://www.mathopenref.com/ellipsefoci.html](http://www.mathopenref.com/ellipsefoci.html)).

1. Whether the ellipse is horizontal or vertical. Remember, we want to the locations of the spaceships to be the foci. And also, they will be facing each other on a horizontal line.

2. Length of the major radius.

3. Length of the minor radius.